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5 STUDENTS IN TOP 10 IN JEE-ADVANCED 2024 OPEN CATEGORY

AIR 1	AIR 4	AIR 5	AIR 6	AIR 9	AIR 10
RAGHAV SHARMA	RHYTHM KEDIA	P KUSHAL KUMAR	RAJDEEP MISHRA	DHRUVIN H DOSHI	A SIDHVIK SUHAS
Ht. No. 242053073*	Ht. No. 247025176*	Ht. No. 246150349	Ht. No. 241016176*	Ht. No. 241106162	Ht. No. 246119101

HIGHLIGHTS

- BELOW 100 — ALL INDIA OPEN CATEGORY RANKS: **30**
- BELOW 500 — ALL INDIA OPEN CATEGORY RANKS: **122**
- BELOW 1000 — ALL INDIA OPEN CATEGORY RANKS: **203**
- BELOW 100 — ALL INDIA CATEGORY RANKS COUNT: **146**
- BELOW 1000 — ALL INDIA CATEGORY RANKS COUNT: **721**
- NUMBER OF QUALIFIED RANKS: **4187+**

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JEE MAIN (JAN) 2025 - SHIFT 2

29-01-2025

Sri Chaitanya IIT Academy., India.

A.P, TELANGANA, KARNATAKA, TAMILNADU, MAHARASHTRA, DELHI, RANCHI

A right Choice for the Real Aspirant

ICON Central Office – Madhapur – Hyderabad

2025_Jee-Main_29-Jan-2025_Shift-02

MATHEMATICS **Max Marks: 100**

(SINGLE CORRECT ANSWER TYPE)

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.

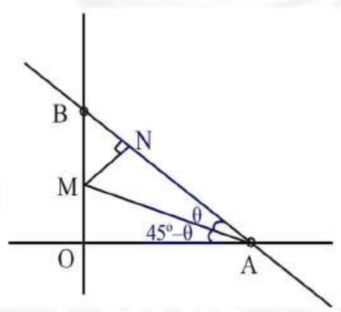
Marking scheme: +4 for correct answer, 0 if not attempted and –1 in all other cases. 1.

1. Let the line $x+y=1$ meet the axes of x and y at A and B , respectively. A right angled triangle AMN is inscribed in the triangle OAB , where O is the origin and the points M and N lie on the lines OB and AB, respectively. If the area of the triangle AMN is $\frac{4}{9}$ of the area of the triangle OAB and $\text{AN}:\text{NB}=\lambda:1$, then the sum of all possible value(s) of is λ :

1) $\frac{1}{2}$ 2) $\frac{13}{6}$ 3) $\frac{5}{2}$ 4) 2

key. (4)

Sol.



Area of $\triangle\text{AOB}=\frac{1}{2}$

Area of $\triangle\text{AMN}=\frac{4}{9}\times\frac{1}{2}=\frac{2}{9}$

Equation of AB is $\text{x}+\text{y}=1$

$\text{OA}=1, \text{AM}=\sec\left(45^\circ-\theta\right)$

$\text{AN}=\sec\left(45^\circ-\theta\right)\cos\theta$

$\text{MN}=\sec\left(45^\circ-\theta\right)\sin\theta$

$\text{Ar}\left(\triangle\text{AMN}\right)=\frac{1}{2}\times\sec^2\left(45^\circ-\theta\right)\sin\theta\cdot\cos\theta=\frac{2}{9}$

$\Rightarrow \tan\theta = 2, \frac{1}{2}$

$\tan\theta = 2$ is rejected

$\frac{AN}{NB} = \frac{\lambda}{1} = \cot\theta = 2$

2. Let the function $f(x) = (x^2 - 1)|x^2 - ax + 2| + \cos|x|$ be not differentiable at the two points $x = \alpha = 2$ and $x = \beta$. Then the distance of the point (α, β) from the line $12x + 5y + 10 = 0$ is equal to

1) 3 2) 4 3) 2 4) 5

Key: 1

Sol. $\cos|x|$ is always differentiable

$\therefore$ we have to check only for $|x^2 - ax + 2|$

$\therefore$ Not differentiable at

$x^2 - ax + 2 = 0$

One root is given, $\alpha = 2$

$\therefore 4 - 2a + 2 = 0$

$a = 3$

$\therefore$ other root $\beta = 1$

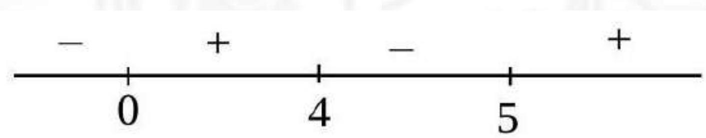
but for $x = 1 f(x)$ is differentiable

3. Let $f(x) = \int_0^x t(t^2 - 9t + 20)dt, 1 \le x \le 5$. If the range of f is $[\alpha, \beta]$, then $4(\alpha + \beta)$ equals

1) 157 2) 253 3) 125 4) 154

key. (1)

Sol. $f'(x) = x^3 - 9x^2 + 20x = x(x-4)(x-5)$



$\therefore \mathbf{f}(\mathbf{x}) = \frac{x^4}{4} - \frac{9x^3}{3} + \frac{20x^2}{2}$

$f(1) = \frac{1}{4} - 3 + 10 = \frac{29}{4} = \alpha$

$f(4) = \frac{256}{4} - 3(64)\Big| + 10(16) = 32 = \beta$

$4(\alpha + \beta) = 4\left(\frac{29}{4} + 32\right) = 157$

4. Let $\alpha, \beta\left(\alpha \neq \beta\right)$ be the value of m, for which the equations

$x+y+z=1; x+2y+4z=m$ and $x+4y+10z=m^2$ have infinitely many solutions. Then the value of $\sum_{n=1}^{10}\left(n^{\alpha}+n^{\beta}\right)$ is equal to:

1) 440 2) 3080 3) 3410 4) 560

Key: 1

Sol :

$$\Delta=\begin{vmatrix}1 & 1 & 1\\ 1 & 2 & 4\\ 1 & 4 & 10\end{vmatrix}=1(20-16)-1(10-4)+1(4-2)$$

$$=4-6+2=0$$

For infinite solutions

$$\Delta_x=\Delta_y=\Delta_z=0$$

$$m^2-3x+2=0$$

$$m=1,2$$

$$\alpha=1, \beta=2$$

$$\therefore \sum_{n=1}^{10}(n^{\alpha}+n^{\beta})=\sum_{n=1}^{10}n^1+\sum_{n=1}^{10}n^2$$

$$=\frac{10(11)}{2}+\frac{10(11)(21)}{6}$$

$$=55+385$$

$$=440$$

5. Let $A=\left[a_{ij}\right]$ be a 2×2 matrix such that $a_{ij}\in\{0,1\}$ for all i and j . Let the random variable X denote the possible values of the determinant of the matrix A . Then, the variance of X is:

1) $\frac{1}{4}$ 2) $\frac{3}{8}$ 3) $\frac{5}{8}$ 4) $\frac{3}{4}$

key. (2)

Sol. $|A|=\begin{vmatrix}a_{11} & a_{12}\\ a_{21} & a_{22}\end{vmatrix}=a_{11}a_{22}-a_{21}a_{12}=\{-1,0,1\}$

x	P_i	P_iX_i	$P_iX_i^2$
-1	$\frac{3}{16}$	$-\frac{3}{16}$	$\frac{3}{16}$
0	$\frac{10}{16}$	0	0
1	$\frac{3}{16}$	$\frac{3}{16}$	$\frac{3}{16}$
		$\sum P_iX_i = 0$	$\sum P_iX_i^2 = \frac{3}{8}$

$\therefore \operatorname{var}(x) = \sum P_iX_i^2 - \left(\sum P_iX_i\right)^2 = \frac{3}{8} - 0 = \frac{3}{8}$

6. If all the words with or without meaning made using all the letters of the word "KANPUR" are arranged as in a dictionary, then the word at 440^{th} position in this arrangement, is :

1) PRNAKU 2) PRKAUN 3) PRNAUK 4) PRKANU

key. (3)

sol:

7. Let a straight line L pass through the point $P(2,-1,3)$ and be perpendicular to the lines $\frac{x-1}{2} = \frac{y+1}{1} = \frac{z-3}{-2}$ and $\frac{x-3}{1} = \frac{y-2}{3} = \frac{z+2}{4}$. If the line L intersects the $yz-$ plane at the point Q, then the distance between the points P and Q is :

1) 2 2) $\sqrt{10}$ 3) 3 4) $2\sqrt{3}$

key. (3)

Sol. Vector parallel to 'L'

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -2 \\ 1 & 3 & 4 \end{vmatrix} = 10\hat{i} - 10\hat{j} + 5\hat{k}$$

$$= 5\left(2\hat{i} - 2\hat{j} + \hat{k}\right)$$

Equation of ' L '

$$\frac{x-2}{2} = \frac{y+1}{-2} = \frac{z-3}{1} = \lambda \text{ (say)}$$

Let $Q(2\lambda+2, -2\lambda-1, \lambda+3)$

$\Rightarrow 2\lambda + 2 = 0 \Rightarrow \lambda = -1$

$\Rightarrow Q(0,1,2)$

$d(P,Q)=3$

A, K, N, P, R, U

[A] .... $= \lfloor 5 = 120$

[K] .... $= \lfloor 5 = 120$

[N] .... $= \lfloor 5 = 120$

[P][A] .... $= \lfloor 4 = 24$

[P][K] .... $= \lfloor 4 = 24$

[P][N] .... $= \lfloor 4 = 24$

[P][R][A] .... $= \lfloor 3 = 6$

[P][R][K][A][N][U] $= 1$

[P][R][K][A][U][N] $= 1$

Total = 440

$\Rightarrow$ 440[th] word

8. The remainder, when 7^{103} is divided by 23 , is equal to:

1) 14 2) 9 3) 17 4) 6

key. (1)

Sol. $7^{103}=7\left(7^{102}\right)=7(343)^{34}=7(345-2)^{34}$

$7^{103}=23\,K_1+7.2^{34}$

Now $7.2^{34}=7.2^{2}\cdot 2^{32}$

$=28.\ (256)^{4}$

$=28(253+3)^{4}$

$\therefore 28\times 81 \Rightarrow (23+5)(69+12)$

$23\,K_2+60$

$\therefore$ Remainder $=14$

9. Let $\mathbf{S}=\mathbf{N}\cup\{0\}$. Define a relation $\mathbf{R}$ from S to $\mathbf{R}$ by :

$$\mathbf{R}=\left\{(x,y):\log_e y = x\log_e\left(\frac{2}{5}\right), x\in S, y\in \mathbf{R}\right\}$$

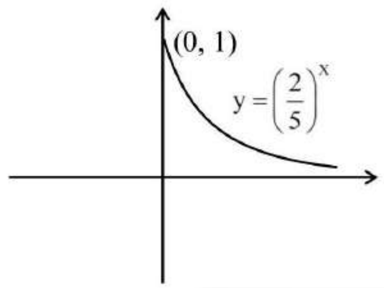
Then, the sum of all the elements in the range of $\mathbf{R}$ is equal to

1) $\frac{3}{2}$ 2) $\frac{5}{3}$ 3) $\frac{10}{9}$ 4) $\frac{5}{2}$

key. (2)

Sol. $S = \{0, 1, 2, 3 \ldots.$

$$\log_e y = x \log_e \left(\frac{2}{5}\right) \Rightarrow y = \left(\frac{2}{5}\right)^x$$



Required

$$\text{Sum} = 1 + \left(\frac{2}{5}\right)^1 + \left(\frac{2}{5}\right)^2 + \left(\frac{2}{5}\right)^3 + \ldots.. - = \frac{1}{1 - \frac{2}{5}} = \frac{5}{3}$$

10. Bag 1 contains 4 white balls and 5 black balls, and Bag 2 contains n white balls and 3 black balls. One ball is drawn randomly from Bag 1 and transferred to Bag 2. A ball is then drawn randomly from Bag 2. If the probability that the ball drawn is white, is 29/45, then n is equal to:

1) 3 2) 4 3) 5 4) 6

Key: 4

Sol:

$\text{Bag } 1 = \{4W, 5B\}$

$\textbf{Bag 2} = \{nW, 3B\}$

$$P\left(\frac{W}{\text{Bag } 2}\right) = \frac{29}{45}$$

$$\Rightarrow P\left(\frac{W}{B_1}\right) \times P\left(\frac{W}{B_2}\right) + P\left(\frac{B}{B_1}\right) \times P\left(\frac{W}{B_2}\right) = \frac{29}{45}$$

$$\frac{4}{9} \times \frac{n+1}{n+4} + \frac{5}{9} \times \frac{n}{n+4} = \frac{29}{45}$$

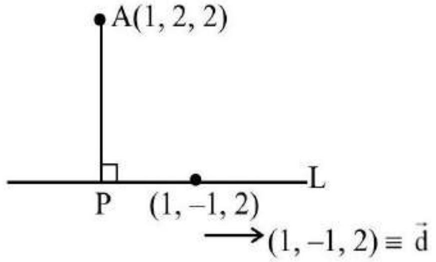
$$\boxed{n = 6}$$

11. Let P be the foot of the perpendicular from the point $(1, 2, 2)$ on the line $L : \frac{x-1}{1} = \frac{y+1}{-1} = \frac{z-2}{2}$. Let the line $\vec{r} = \left(-\hat{i} + \hat{j} - 2\hat{k}\right) + \lambda\left(\hat{i} - \hat{j} + \hat{k}\right), \lambda \in \mathbf{R}$, intersect the line L at Q. Then $2(PQ)^2$ is equal to:

1) 27 2) 25 3) 29 4) 19

key. (1)

Sol.



$L: \frac{x-1}{1} = \frac{y+1}{-1} = \frac{z-2}{2} = \mu$

$P(\mu+1, -\mu-1, 2\mu+2)$

$\overrightarrow{AP} \cdot \vec{d} = 0 \Rightarrow (\mu, -\mu-3, 2\mu) \cdot (1, -1, 2) = 0$

$\Rightarrow \mu + \mu + 3 + 4\mu = 0 \Rightarrow \mu = -\frac{1}{2}$

$\therefore P\left(\frac{-1}{2}+1, +\frac{1}{2}-1, 2\left(\frac{-1}{2}\right)+2\right)$

$P\left(\frac{1}{2}, \frac{-1}{2}, 1\right)$

Now general pt. on L_2 is $Q(-1+\lambda, 1-\lambda, -2+\lambda)$

Equate it with general pt of L

$\mu+1=-1+\lambda$	$-\mu-1=1-\lambda$	$2\mu+2=-2+\lambda$
$\mu=\lambda-2$	$\mu=\lambda-2$	$\downarrow$

$2(\lambda-2)+2=-2+\lambda$

$2\lambda-4+2=-2+\lambda$

$\therefore \mu=-2, \lambda=0 \therefore Q \equiv (-1, 1-2)$

$P\left(\frac{1}{2}, \frac{-1}{2}, 1\right)$ and $Q(-1, 1, -2)$

$PQ = \sqrt{\left(\frac{1}{2}+1\right)^2 + \left(\frac{-1}{2}-1\right)^2 + (1+2)^2} \quad = \sqrt{\frac{9}{4}+\frac{9}{4}+9} = \sqrt{\frac{54}{4}} \quad \therefore 2(PQ)^2 = 2\left(\frac{54}{4}\right) = 27$

12. Let the area enclosed between the curves $|y| = 1 - x^2$ and $x^2 + y^2 = 1$ be α. If $9\alpha = \beta\pi + \gamma; \beta, \gamma$ are integers, then the value of $|\beta - \gamma|$ equals

1) 27 2) 18 3) 15 4)33

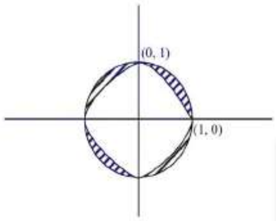
key. (4)

Sol. $C_1 : |y| = 1 - x^2$

$C_2 : x^2 + y^2 = 1 \therefore$ Required Area

$= \alpha = 4\left[\right.$ Area of circle in 1^{st} quad. $\left.-\int_0^1\left(1-x^2\right)dx\right] = 4\left[\frac{\pi}{4}-\left[x-\frac{x^3}{3}\right]_0^1\right]$

$\alpha = \pi - \frac{8}{3} \therefore 3\alpha = 3\pi - 8 \therefore 9\alpha = 9\pi - 24 \therefore \beta = 9, \gamma = -24 \therefore |\beta - \gamma| = 33$

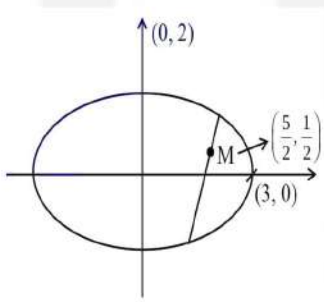


13. If $\alpha x + \beta y = 109$ is the equation of the chord of the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$, whose mid point is $\left(\frac{5}{2}, \frac{1}{2}\right)$, then $\alpha + \beta$ is equal to

1) 37 2) 46 3) 58 4) 72

key. (3)

Sol.



Equation of chord $T = S_1$ $\frac{5}{2}\left(\frac{x}{9}\right) + \frac{1}{2}\left(\frac{y}{4}\right) = \frac{25}{36} + \frac{1}{16}$

$\Rightarrow \frac{5x}{18} + \frac{y}{8} = \frac{100+9}{144} = \frac{109}{144}$ $\Rightarrow 40x + 18y = 109$

$\Rightarrow \alpha = 40, \beta = 18$

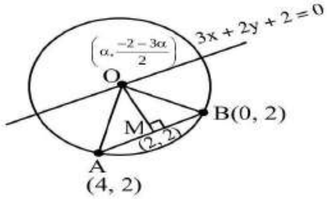
$\Rightarrow \alpha + \beta = 58$

14. Let a circle C pass through the points $(4,2)$ and $(0, 2)$, and its centre lie on $3x + 2y + 2 = 0$. Then the length of the chord, of the circle C, whose midpoint is $(1,2)$, is:

1) $\sqrt{3}$ 2) $2\sqrt{3}$ 3) $4\sqrt{2}$ 4) $2\sqrt{2}$

key. (2)

Sol.



$M_{AB} = 0 \Rightarrow OM$ is vertical $\Rightarrow \alpha = 2 \therefore$ Centre $(0) \equiv (2,-4)$

$r = OA = \sqrt{(2-4)^2 + (2+4)^2} = \sqrt{40}$

mid point of chord is $N \equiv (1,2) \therefore ON = \sqrt{37}$

$\therefore$ length of chord $= 2\sqrt{r^2 - (ON)^2} \qquad = 2\sqrt{40-37} = 2\sqrt{3}$

15. If for the solution curve $y = f(x)$ of the differential equation $\frac{dy}{dx} + (\tan x)y = \frac{2+\sec x}{(1+2\sec x)^2}$, $x \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right), f\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{10}$, then $f\left(\frac{\pi}{4}\right)$ is equal to:

1) $\frac{9\sqrt{3}+3}{10(4+\sqrt{3})}$ 2) $\frac{\sqrt{3}+1}{10(4+\sqrt{3})}$ 3) $\frac{5-\sqrt{3}}{2\sqrt{2}}$ 4) $\frac{4-\sqrt{2}}{14}$

Key: (4)

Sol: If $e^{\int \tan x dx} = e^{\ln(\sec x)} = \sec x \therefore \mathbf{y} \cdot \sec x = \int \left\{\frac{2+\sec x}{(1+2\sec x)^2}\right\} \sec x dx$

$= \int \frac{2\cos x + 1}{(\cos x + 2)^2} dx$ Let $\cos x = \frac{1-t^2}{1+t^2} = \int \frac{2\left(\frac{1-t^2}{1+t^2}\right)+1}{\left(\frac{1-t^2}{1+t^2}+2\right)^2} 2dt$

$= \int \frac{2-2t^2+1+t^2}{\left(1-t^2+2+2t^2\right)^2} \times 2dt = 2\int \frac{3-t^2}{\left(t^2+3\right)^2} dt$

Let $t + \frac{3}{t} = u \left(1 - \frac{3}{t^2}\right) dt = du = -2\int \frac{du}{u^2} \qquad y \cdot (\sec x) = \frac{2}{u} + c$

At $x = \frac{\pi}{3}, t = \tan\frac{x}{2} = \frac{1}{\sqrt{3}}$

$2. \frac{\sqrt{3}}{10} = \frac{2}{\frac{1}{\sqrt{3}} + 3\sqrt{3}} + c$

$2. \frac{\sqrt{3}}{10} = \frac{2\sqrt{3}}{10} + c \Rightarrow C = 0$

At $x=\frac{\pi}{4}, t=\tan\frac{x}{2}=\sqrt{2}-1$

$$\therefore y\cdot\sqrt{2}=\frac{2}{\sqrt{2}-1+\frac{3}{\sqrt{2}-1}}$$

$$y.\ \sqrt{2}=\frac{2\left(\sqrt{2}-1\right)}{6-2\sqrt{2}}$$

$$y=\frac{\sqrt{2}\left(\sqrt{2}-1\right)}{2\left(3-\sqrt{2}\right)}=\frac{1}{\sqrt{2}}\times\frac{2\sqrt{2}-1}{7}=\frac{4-\sqrt{2}}{14}$$

16. If the domain of the function $\log_5\left(18x-x^2-77\right)$ is (α,β) and the domain of the function $\log_{(x-1)}\left(\frac{2x^2+3x-2}{x^2-3x-4}\right)$ is (γ,δ), then $\alpha^2+\beta^2+\gamma^2$ is equal to

1) 195 2) 174 3) 186 4) 179

key. (3)

Sol. $f_1(x)=\log_5\left(18x-x^2-77\right)$

$\therefore 18x-x^2-77>0$

$x^2-18x+77<0$

$x\in(7,11)\ \alpha=7,\beta=11$

$$f_2(x)=\log_{(x-1)}\left(\frac{2x^2+3x-2}{x^2-3x-4}\right)$$

$$\therefore\ x-1>0, x-1\neq 1, \frac{2x^2+3x-2}{x^2-3x-4}>0$$

$$x>1, x\neq 2, \frac{(2x-1)(x+2)}{(x-4)(x+1)}>0$$

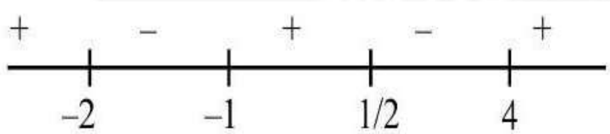
$x>1, x\neq 2$,

$\therefore x\in(4,\infty)$

$\therefore \gamma=4$

$\therefore \alpha^2+\beta^2+\gamma^2=49+121+16$

$=186$



17. Let â be a unit vector perpendicular to the vectors $\vec{b}=\hat{i}-2\hat{j}+3\hat{k}$ and $\vec{c}=2\hat{i}+3\hat{j}-\hat{k}$, and makes an angle of $\cos^{-1}\left(-\frac{1}{3}\right)$ with the vector $\hat{i}+\hat{j}+\hat{k}$. If $\hat{a}$ makes an angle of $\frac{\pi}{3}$ with the vector $\hat{i}+\alpha\hat{j}+\hat{k}$, then the value of α is :

1) $-\sqrt{3}$ 2) $\sqrt{6}$ 3) $-\sqrt{6}$ 4) $\sqrt{3}$

key. (3)

Sol. Let $\vec{v}=\hat{i}+\hat{j}+\hat{k}$

$$\vec{b}\times\vec{c}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -2 & 3\\ 2 & 3 & -1\end{vmatrix}$$

$$=-7\hat{i}+7\hat{j}+7\hat{k}$$

$$=-7\left(\hat{i}-\hat{j}-\hat{k}\right)$$

Now $\hat{a}=\frac{\hat{i}-\hat{j}-\hat{k}}{\sqrt{3}}$ or $\frac{-\hat{i}+\hat{j}+\hat{k}}{\sqrt{3}}$

$\cos\theta=\frac{\hat{a}\cdot\vec{v}}{|\vec{v}|}=\frac{1-1-1}{\sqrt{3}\sqrt{3}}=\frac{-1}{3}$ $\cos\theta=\frac{\hat{a}\cdot\vec{v}}{|\vec{v}|}=\frac{-1+1+1}{3}=\frac{1}{3}$

(rejected) $\Rightarrow\hat{a}=\frac{\hat{i}-\hat{j}-\hat{k}}{\sqrt{3}}$

Now $\cos\frac{\pi}{3}=\frac{\hat{a}\cdot\left(\hat{i}+\alpha\hat{j}+\hat{k}\right)}{\sqrt{1+\alpha^2+1}}\Rightarrow\frac{1}{2}=\frac{1-\alpha-1}{\sqrt{3}\sqrt{\alpha^2+2}}$

$\Rightarrow\frac{\sqrt{3}}{2}\sqrt{\alpha^2+2}=-\alpha\,(\therefore\alpha<0)$

$3\alpha^2+6=4\alpha^2\Rightarrow\alpha=-\sqrt{6}$

18. Let $A=\left[a_{ij}\right]$ be a matrix of order 3×3, with $a_{ij}=(\sqrt{2})^{i+j}$. If the sum of all the elements in the third row of A^2 is $\alpha+\beta\sqrt{2},\alpha,\beta\in\mathbf{Z}$, then $\alpha+\beta$ is equal to

1) 280 2) 168 3) 210 4) 224

key. (4)

Sol. $$A=\begin{bmatrix}(\sqrt{2})^2 & (\sqrt{2})^3 & (\sqrt{2})^4\\ (\sqrt{2})^3 & (\sqrt{2})^4 & (\sqrt{2})^5\\ (\sqrt{2})^4 & (\sqrt{2})^5 & (\sqrt{2})^6\end{bmatrix}$$

$$A=\begin{bmatrix}2 & 2\sqrt{2} & 4\\ 2\sqrt{2} & 4 & 4\sqrt{2}\\ 4 & 4\sqrt{2} & 8\end{bmatrix}$$

$$A^2=2^2\begin{bmatrix}1 & \sqrt{2} & 2\\ \sqrt{2} & 2 & 2\sqrt{2}\\ 2 & 2\sqrt{2} & 4\end{bmatrix}\begin{bmatrix}1 & \sqrt{2} & 2\\ \sqrt{2} & 2 & 2\sqrt{2}\\ 2 & 2\sqrt{2} & 4\end{bmatrix}$$

$$= 4\begin{bmatrix} - & - & - \\ - & - & - \\ (2+4+8) & (2\sqrt{2}+4\sqrt{2}+8\sqrt{2}) & (4+8+16) \end{bmatrix}$$

Sum of elements of 3^{rd} row $= 4\left(14+14\sqrt{2}+28\right)$

$= \quad 4\left(42+14\sqrt{2}\right)$

$= \quad 168+56\sqrt{2}$

$\alpha+\beta\sqrt{2}$

$\therefore \alpha \quad \alpha\beta = 168+56 = 224$

19. If the set of all $a \in R$, for which the equation $2x^2+(a-5)x+15=3a$ has no real root, is the interval (α,β), and $X=\{x\in Z:\alpha<x<\beta\}$, then $\sum_{x\in X} x^2$ is equal to:

1) 2109 2) 2129 3) 2139 4) 2119

Key: (3)

Sol: $(a-5)^2-8(15-3a)<0$

$a^2+14a+25-120<0$ $a^2+14a-95<0$

$(a+19)(a-5)<0$ $a\in(-19,5)$ $\therefore -19<x<5$

$\therefore \sum_{x\in X} x^2 = \left(1^2+2^2+\ldots+4^2\right)+\left(1^2+2^2+\ldots+18^2\right)$

$= \frac{4\times5\times9}{6}+\frac{18\times19\times37}{6} = 30+2109 = 2139$

20. If $\sin x+\sin^2 x=1, x\in\left(0,\frac{\pi}{2}\right)$, then $\left(\cos^{12}x+\tan^{12}x\right)+3\left(\cos^{10}x+\tan^{10}x+\cos^8 x+\tan^8 x\right)$ $+\left(\cos^6 x+\tan^6 x\right)$ is equal to

1) 4 2) 3 3) 2 4) 1

key. (3)

Sol. $\sin x+\sin^2 x=1 \Rightarrow \sin x=\cos^2 x \Rightarrow \tan x=\cos x$

$\therefore$ Given expression

$= 2\cos^{12}x+6\left[\cos^{10}x+\cos^8 x\right]+2\cos^6 x$

$= 2\left[\sin^6 x+3\sin^5 x+3\sin^4 x+\sin^3 x\right]$

$= 2\sin^3 x\left[(\sin x+1)^3\right]$

$= 2\left[\sin^2 x+\sin x\right]^3 = 2$

(NUMERICAL VALUE TYPE)

This section contains 10 questions. Each question is numerical value type. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to second decimal place. (e.g. 6.25, 7.00, 0.33, 30, 30.27, 127.30). Attempt any five questions out of 10.

Marking scheme: +4 for correct answer, 0 if not attempted and 0 in all other cases.

21. Let $y^2 = 12x$ the parabola and S be its focus. Let PQ be a focal chord of the parabola such that (SP) $(SQ) = \frac{147}{4}$. Let C be the circle described taking PQ as a diameter. If the equation of a circle C is $64x^2 + 64y^2 - \alpha x - 64\sqrt{3}y = \beta$, then $\beta - \alpha$ is equal to

Key (1328)

Sol. $y^2 = 12x$ $a = 3$ $SP \times SQ = \frac{147}{4}$ Let $P(3t^2, 6t)$ and $t_1 t_2 = -1$

(ends of focal chord)

So, $Q\left(\frac{3}{t^2}, \frac{-6}{t}\right)$ $S(3,0)$ $SP \times SQ = PM_1 \times QM_2$

(dist. from directrix) $= (3 + 3t^2)\left(3 + \frac{3}{t^2}\right) = \frac{147}{4} \Rightarrow \frac{(1+t^2)^2}{t^2} = \frac{49}{12}$

$t^2 = \frac{3}{4}, \frac{4}{3}$ $t = \pm\frac{\sqrt{3}}{2}, \pm\frac{2}{\sqrt{3}}$

considering $t = \frac{-\sqrt{3}}{2}$ $P\left(\frac{9}{4}, -3\sqrt{3}\right)$ and $Q(4, 4\sqrt{3})$

Hence, diametric circle:

$(x-4)\left(x - \frac{9}{4}\right) + (y + 3\sqrt{3})(y - 4\sqrt{3}) = 0$

$\Rightarrow x^2 + y^2 - \frac{25}{4}x - \sqrt{3}y - 27 = 0 \Rightarrow \alpha = 400, \beta = 1728$ $\beta - \alpha = 1328$

22. Let $a_1, a_2, \ldots, a_{2024}$ be an Arithmetic Progression such that $a_1 + (a_5 + a_{10} + a_{15} + \ldots + a_{2020}) + a_{2024} = 2233$. Then $a_1 + a_2 + a_3 + \ldots + a_{2024}$ is equal to

Key: (11132)

Sol :

$a_1 + a_5 + a_{10} + \ldots .. + a_{2020} + a_{2024} = 2233$

In an A.P. the sum of terms equidistant from ends is equal.

$a_1 + a_{2024} = a_5 + a_{2020} = a_{10} + a_{2015} \ldots ..$

$\Rightarrow$ 203 pairs

$\Rightarrow 203(a_1 + a_{2024}) = 2233$

Hence,

$S_{2024} = \frac{2024}{2}(a_1 + a_{2024})$

$= 1012 \times 11$

$= 11132$

23. If $24\int_0^{\frac{\pi}{4}}\left(\sin\left|4x - \frac{\pi}{12}\right| + [2\sin x]\right)dx = 2\pi + \alpha$, where [•] denotes the greatest integer function, then α is equal to

key. (12)

Sol. $= 24\int_0^{\frac{\pi}{48}} -\sin\left(4x - \frac{\pi}{12}\right) + \int_{\pi/48}^{\pi/4}\sin\left(4x - \frac{\pi}{12}\right)$

$$+\int_0^{\frac{\pi}{6}}[0]dx+\int_{\pi/6}^{\pi/4}\left[2\sin_{\downarrow} x\right]dx$$

$$= \quad 24\left[\frac{\left(1-\cos\frac{\pi}{12}\right)}{4}-\frac{\left(-\cos\frac{\pi}{12}-1\right)}{4}\right]+\frac{\pi}{4}-\frac{\pi}{6}$$

$$=24\left(\frac{1}{2}+\frac{\pi}{12}\right)=2\pi+12 \quad \alpha=12$$

24. Let integers $a,b\in[-3,3]$ be such that $a+b\neq 0$. Then the number of all possible ordered pairs (a, b), for which $\left|\frac{z-a}{z+b}\right|=1$ and $\begin{vmatrix} z+1 & \omega & \omega^2 \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix}=1, z\in C$, where ω and ω^2 are the roots of $x^2+x+1=0$, is equal to

Key: (10)

SOL:

$a, b \in I, -3\le a, b\le 3, a+b\neq 0$

$|z-a|=|z+b|$

$$\begin{vmatrix} z+1 & \omega & \omega^2 \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix}=1$$

$$\Rightarrow \begin{vmatrix} z & z & z \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix}=1$$

$$\Rightarrow z\begin{vmatrix} 1 & 1 & 1 \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix}=1$$

$$\Rightarrow z\begin{vmatrix} 1 & 0 & 0 \\ \omega & z+\omega^2-\omega & 1-\omega \\ \omega^2 & 1-\omega^2 & z+\omega-\omega^2 \end{vmatrix}=1$$

$\Rightarrow z^3=1$

$\Rightarrow z=\omega, \omega^2, 1$

Now

$|1-a|=|1+b|$

$\Rightarrow$ 10 pairs

25. If $\lim_{t\to 0}\left(\int_0^1(3x+5)^t dx\right)^{\frac{1}{t}}=\frac{\alpha}{5e}\left(\frac{8}{5}\right)^{\frac{2}{3}}$, then α is equal to

Key: 64

Sol:

1^∞ form

$$\text{Now } L=e^{t\to 0}\frac{1}{t}\left(\left.\frac{(3x+5)^{t+1}}{3(t+1)}\right|_0^1-1\right)$$

$$=e^{t\to 0}\frac{8^{t+1}-5^{t+1}-3t-3}{3t(t+1)}$$

$$=e^{\frac{8\ell n8-5\ell n5-3}{3}}$$

$$=\left(\frac{8}{5}\right)^{2/3}\left(\frac{64}{5}\right)=\frac{\alpha}{5e}\left(\frac{8}{5}\right)^{2/3}$$

On comparing

$\alpha=64$

PHYSICS **Max Marks: 100**

(SINGLE CORRECT ANSWER TYPE)

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and –1 in all other cases.

26. A convex lens mode of glass (refractive index = 1.5) has focal length 24 cm in air. When it is totally immersed in water (refractive index $=1.33$), its focal length changes to

(1) 72 cm (2) 96 cm (3) 24 cm (4) 48 cm

key. (2)

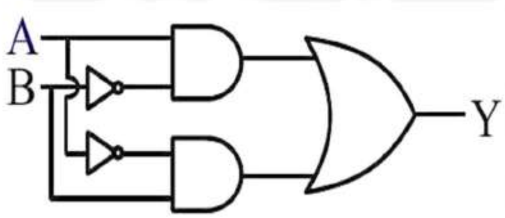
Sol. $\frac{1}{8}=\left(\frac{\mu_\ell}{\mu_s}-1\right)\left[\frac{1}{R_1}-\frac{1}{R_2}\right]$

$\frac{1}{24}=(1.5-1)\left[\frac{2}{R}\right]$ $\frac{1}{f'}=\left(\frac{1.5}{1.33}-1\right)\left(\frac{2}{R}\right)$ $\frac{1}{f'}=\left(\frac{1.5\times 3}{4}-1\right)\frac{2}{R}$

(i) divided by (ii) $\frac{f'}{24}=4$ $f'=96\text{ cm}$

Key (2)

27. The truth table for the circuit given below is :



1)

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

2)

A	B	Y
0	0	0
1	0	0
1	1	0
0	1	1

3)

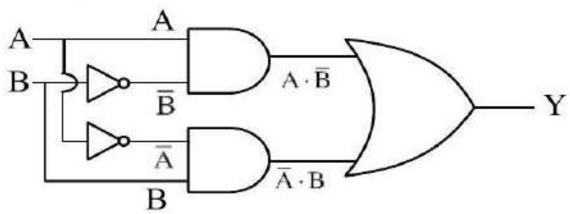
A	B	Y
0	0	0
1	0	1
0	1	0
1	1	0

4)

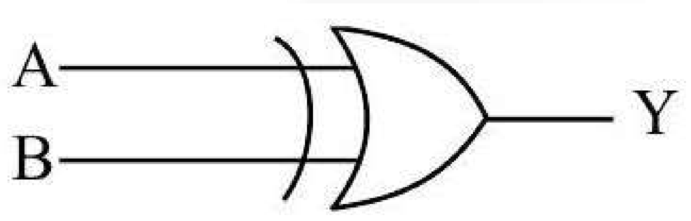
A	B	Y
0	0	0
1	1	1
1	0	1
0	1	1

Key (1)

Sol.



$Y = A\cdot\overline{B} + \overline{A}\cdot B$



XOR (Exclusive OR)

28. Two bodies A and B of equal mass are suspended from two massless springs of spring constant k_1 and k_2, respectively. If the bodies oscillate vertically such that their amplitudes are equal, the ratio of the maximum velocity of A to the maximum velocity of B is

(1) $\sqrt{\frac{k_1}{k_2}}$ (2) $\frac{k_1}{k_2}$ (3) $\frac{k_2}{k_1}$ (4) $\sqrt{\frac{k_2}{k_1}}$

Key (1)

Sol. $V_1 = A_1\omega_1 \quad V_2 = A_2\omega_2 \quad A_1 = A_2$

$$\frac{V_1}{V_2} = \frac{\omega_1}{\omega_2} = \frac{\sqrt{\frac{K_1}{m}}}{\sqrt{\frac{K_2}{m}}} \quad \frac{V_1}{V_2} = \sqrt{\frac{K_1}{K_2}}$$

29. Match List-I with List-II.

	List-I		List-II
(A)	Young's Modulus	(I)	$ML^{-1}T^{-1}$
(B)	Torque	(II)	$ML^{-1}T^{-2}$
(C)	Coefficient of Viscosity	(III)	$M^{-1}L^{3}T^{-2}$
(D)	Gravitational Constant	(IV)	$ML^{2}T^{-2}$

Choose the correct answer from the options given below :

(1) (A)-(I), (B)-(III), (C)-(II), (D)-(IV)

(2) (A)-(II), (B)-(I), (C)-(IV), (D)-(III)

(3) (A)-(IV), (B)-(II), (C)-(III), (D)-(I)

(4) (A)-(II), (B)-(IV), (C)-(I), (D)-(III)

Key (4)

Sol. (A) $[Y] = \frac{F}{A\left(\frac{\Delta\ell}{\ell}\right)} \Rightarrow \frac{MLT^{-2}}{L^2} = ML^{-1}\ T^{-2}$ (II)

(B) Torque $(\vec{\tau}) = \vec{r} \times \vec{F}$

$(\vec{\tau}) = L \times MLT^{-2} = ML^2\ T^{-2}\ (IV)$

(C) Coefficient of viscosity $\Rightarrow F = \eta A \frac{dV}{dt}$

$\eta \to Pa \cdot sec$

$[\eta] = \frac{MLT^{-2}}{L^2} \times T = ML^{-1}\ T^{-1}\ (I)$

(D) Gravitational constant (G)

$F = \frac{GM_1M_2}{r^2}$

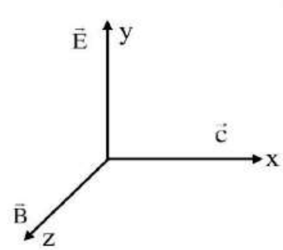
$[G] = \frac{F \cdot r^2}{m_1\ m_2} = \frac{MLT^{-2} \times L^2}{M^2} = M^{-1}\ L^3\ T^{-2}$ (III)

30. A plane electromagnetic wave propagates along the $+x$ direction in free space. The components of the electric field, $\vec{E}$ and magnetic field, $\vec{B}$ vectors associated with the wave in Cartesian frame are :

(1) E_y, B_x (2) E_y, B_z (3) E_x, B_y (4) E_z, B_y

Key (2)

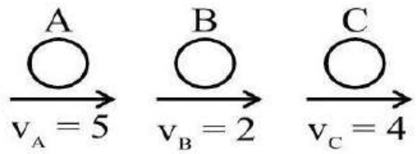
Sol.



Direction of propogation $= \vec{E} \times \vec{B}$

31. Given below are two statements. One is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A) :



Three identical spheres of same mass undergo one dimensional motion as shown in figure with initial velocities $v_A = 5\ m/s, v_B = 2\ m/s, v_C = 4\ m/s$ If we wait sufficiently long for elastic collision to happen, then $v_A = 4\ m/s, v_B = 2\ m/s, v_C = 5\ m/s$ will be the final velocities.

Reason (R) : In an elastic collision between identical masses, two objects exchange their velocities.

In the light of the above statements, choose the correct answer from the options given below :

(1) Both (A) and (R) are true but (R) is NOT the correct explanation of (A)

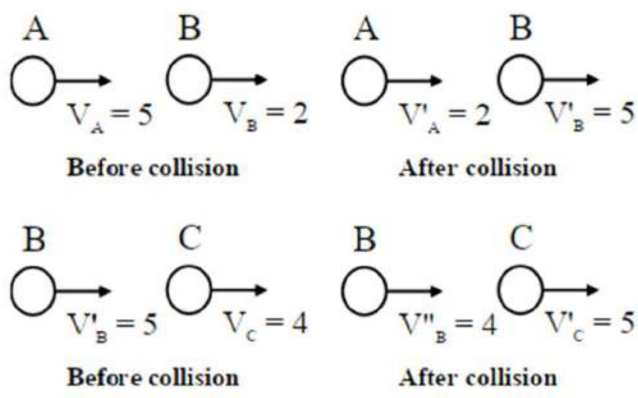
(2) (A) is true but (R) is false

(3) Both (A) and (R) are true and (R) is the correct explanation of (A).

(4) (A) is false but (R) is true

Key (4)

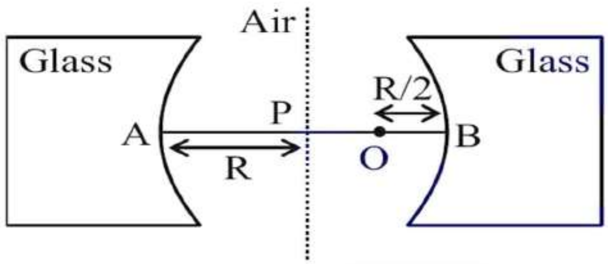
Sol. In elastic collision for same mass, velocities interchange



$V'_A = 2\ m/s\ \ V'_B = 4\ m/s\ \ V'_C = 5\ m/s$

Key (4)

32.



Two concave refracting surfaces of equal radii of curvature and refractive index 1.5 face each other in air as shown in figure. A point object O is placed midway, between P and B. The separation between the images of O , formed by each refracting surface is :

(1) 0.214 R (2) 0.114 R (3) 0.411 R (4) 0.124 R

Key (2)

Sol. For B

$$\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}$$

$$\frac{1.5}{V}+\frac{1}{\frac{R}{2}}=\frac{0.5}{-R}$$

$$\frac{1.5}{V}=-\frac{1}{2R}-\frac{2}{R}$$

$$\frac{1.5}{V}=\frac{-5}{2R}\Rightarrow V_B=-0.6R$$

For A

$$\frac{1.5}{V}+\frac{2}{3R}=\frac{0.5}{-R}$$

$$\frac{1.5}{V}=-\frac{1}{2R}-\frac{2}{3R}$$

$$\frac{1.5}{V}=-\frac{7}{6R}$$

$$V_A=-\frac{9}{7}R$$

Distance between images

$$=2R-\left(0.6R+\frac{9}{7}R\right)=0.114R$$

option (2)

33. **Match List-I with List-II**.

	List-I		List-II
(A)	Magnetic induction	(I)	Ampere meter ^{2}
(B)	Magnetic intensity	(II)	Weber
(C)	Magnetic flux	(III)	Gauss
(D)	Magnetic moment	(IV)	Ampere meter

Choose the correct answer from the options given below :

(1) (A)-(III), (B)-(IV), (C)-(I), (D)-(II)

(2) (A)-(III), (B)-(IV), (C)-(II), (D)-(I)

(3) (A)-(I), (B)-(II), (C)-(III), (D)-(IV)

(4) (A)-(III), (B)-(II), (C)-(I), (D)-(IV)

Key (2)

Sol. (A) Magnetic induction $\rightarrow$ Gauss (III)

(B) Magnetic intensity

$\left(H = \frac{B}{\mu}\right) \rightarrow$ Ampere / meter (IV)

(C) Magnetic flux $\rightarrow$ Weber (Wb) (II)

(D) Magnetic moment $\rightarrow$ Ampere-meter ^{2}

$\left(\vec{M} = i\vec{A}\right)$

Note : None of the option(s) are correct but if we need to choose most appropriate option then the answer is (2)

34. A sand dropper drops sand of mass $m(t)$ on a conveyer belt at a rate proportional to the square root of speed (v) of the belt, i.e. $\frac{dm}{dt} \propto \sqrt{v}$. If P is the power delivered to run the belt at constant speed then which of the following relationship is true?

(1) $P^2 \propto v^3$ (2) $P \propto \sqrt{V}$ (3) $P \propto v$ (4) $P^2 \propto v^5$

Key (4)

Sol. Power $=\vec{F}\cdot\vec{V}$

$$F=\frac{dp}{dt}[p=mv]$$

$$F=\left(\frac{dm}{dt}\right)v=C\left(\sqrt{v}\right)v$$

$$F=Cv^{\frac{3}{2}}$$

Power $=C\left(v^{3/2}\right)v=Cv^{5/2}$

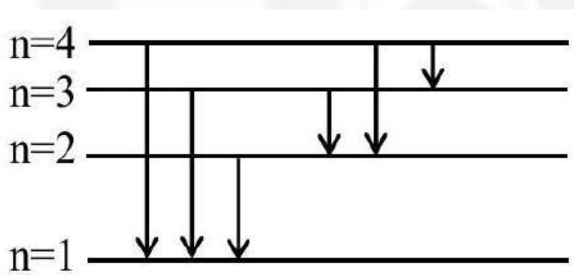
$p^2\propto v^5$

35. The number of spectral lines emitted by atomic hydrogen that is in the 4^{th} energy level, is

(1) 6 (2) 0 (3) 3 (4) 1

Key (1)

Sol



Total possible transition $=6$

36. The difference of temperature in a material can convert heat energy into electrical energy. To harvest the heat energy, the material should have

(1) low thermal conductivity and low electrical conductivity

(2) high thermal conductivity and high electrical conductivity

(3) low thermal conductivity and high electrical conductivity

(4) high thermal conductivity and low electrical conductivity

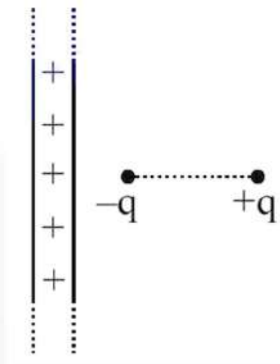
key. (3)

Sol. See-back effect

Low thermal conductivity

High electrical conductivity

37. An electric dipole is placed at a distance of 2 cm from an infinite plane sheet having positive charge density σ_0. Choose the correct option from the following.\



(1) Torque on dipole is zero and net force is directed away from the sheet.

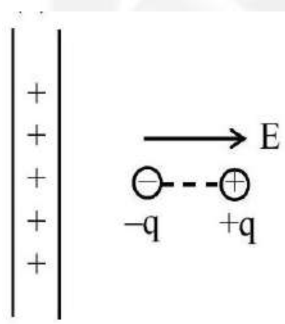
(2) Torque on dipole is zero and net force acts towards the sheet.

(3) Potential energy of dipole is minimum and torque is zero.

(4) Potential energy and torque both are maximum

key. (3)

Sol.



Here $E = \frac{\sigma}{2\epsilon_0}, \vec{\tau} = \vec{P} \times \vec{E}$ $\vec{\tau} = 0$

$U = -\vec{P} \cdot \vec{E}$ $U \to$ minimum

38. In an experiment with photoelectric effect, the stopping potential.

(1) increases with increase in the wavelength of the incident light

(2) increases with increase in the intensity of the incident light

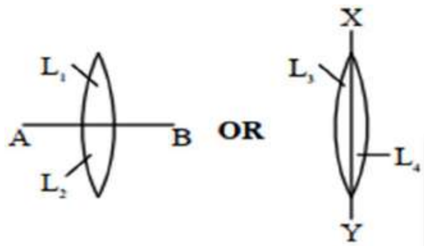
(3) is $\left(\frac{1}{e}\right)$ times the maximum kinetic energy of the emitted photoelectrons

(4) decreases with increase in the intensity of the incident light

key. (3)

Sol. $\frac{hC}{\lambda} = W + eV_S$ $\frac{hC}{\lambda} = W + (K_{max}) \therefore V_S = \frac{K_{max}}{e}$

39. Two identical symmetric double convex lenses of focal length f are cut into two equal parts L_1, L_2 by AB plane and L_3, L_4 by XY plane as shown in figure respectively. The ratio of focal lengths of lenses L_1 and L_3 is



(1) $1:4$ (2) $1:1$ (3) $2:1$ (4) $1:2$

key. (4)

Sol. $f_{L_1} = f_{L_2} = f \quad f_{L_3} = f_{L_4} = 2f \therefore f_{L_1} : f_{L_3} = 1:2$

40. Given below are two statements. One is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A) : With the increase in the pressure of an ideal gas, the volume falls off more rapidly in an isothermal process in comparison to the adiabatic process.

Reason (R) : In isothermal process, PV = constant, while in adiabatic process PV^{γ} = constant. Here γ is the ratio of specific heats, P is the pressure and V is the volume of the ideal gas.

In the light of the above statements, choose the correct answer from the options given below :

(1) Both (A) and (R) are true but (R) is NOT the correct explanation of (A)

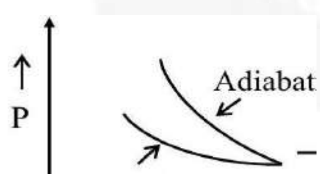
(2) (A) is true but (**R**) is false

(3) Both (A) and (R) are true and (R) is the correct explanation of (A).

(4) (A) is false but (R) is true

Key (3)

Sol.



Pressure increases, Isothermal

$$\left(\frac{dP}{dV}\right)_{Adiabatic} > \left(\frac{dP}{dV}\right)_{Isothermal}$$

41. A point charge causes an electric flux of $-2\times10^4\ \text{Nm}^2\text{C}^{-1}$ to pass through a spherical Gaussian surface of 8.0 cm radius, centred on the charge. The value of the point charge is (Given $\epsilon_0 = 8.85\times10^{-12}\text{C}^2\ \text{N}^{-1}\ \text{m}^{-2}$)

(1) $-17.7\times10^{-8}\text{C}$ (2) $-15.7\times10^{-8}\text{C}$ (3) $17.7\times10^{-8}\text{C}$ (4) $15.7\times10^{-8}\text{C}$

key. (1)

Sol. $\phi = -2\times10^4 \dfrac{\text{Nm}^2}{\text{C}}$

$r = 8.0\ \text{cm}$

$\phi = \dfrac{q}{\epsilon_0} \Rightarrow q = \epsilon_0\phi$

$= \left(8.85\times10^{-12}\right)\times\left(-2\times10^4\right)$

$q = -17.7\times10^{-8}\text{C}$

42. A cup of coffee cools from 90°C to 80°C in t minutes when the room temperature is 20°C. The time taken by the similar cup of coffee to cool from 80°C to 60°C at the same room temperature is :

(1) $\dfrac{13}{5}t$ (2) $\dfrac{10}{13}t$ (3) $\dfrac{13}{10}t$ (4) $\dfrac{5}{13}t$

Key (1)

Sol. By using average form of Newton's law of cooling

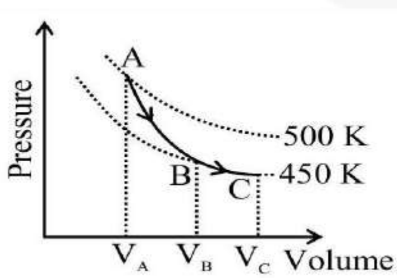
$$\frac{90-80}{t} = k\left(\frac{90+80}{2} - 20\right)$$

$$\frac{80-60}{t'} = k\left(\frac{80+60}{2} - 20\right)$$

(i)/(ii)

$$\frac{10\times t'}{t\times20} = \frac{65}{50} \quad t' = \frac{65}{50}\times2t = \frac{65}{25}t = \frac{13}{5}t$$

43.



A poly-atomic molecule $(C_v = 3R, C_p = 4R$, where R is gas constant) goes from phase space point $A(P_A = 10^5\ Pa,\ V_A = 4\times10^{-6}\ m^3)$ to point $B(P_B = 5\ \times10^4\ Pa,\ V_B = 6\times10^{-6}\ m^3)$ to point $C(P_C = 10^4\ Pa$, $V_c = 8\times10^{-6}\ m^3$). A to B is an adiabatic path and B to C is an isothermal path.

The net heat absorbed per unit mole by the system is :

(1) $500R(\ln3+\ln4)$ (2) $450R(\ln4-\ln3)$ (3) $500R\ln2$

(4) $400R\ln4$

Key (2)

Sol. $\Delta Q_{AB} = 0$ adiabatic

$$\Delta Q_{BC} = \Delta W_{BC}$$

$$= nRT\ell\left(\frac{V_C}{V_B}\right) = 450R\ell n\left(\frac{8\times10^{-6}}{6\times10^{-6}}\right)$$

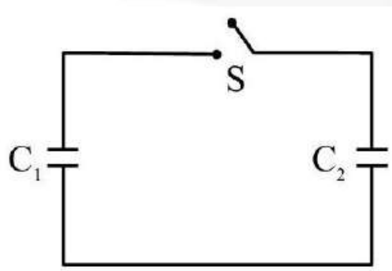
$$= 450R\ln\left(\frac{4}{3}\right) = 450R(\ln4-\ln3)$$

$$\therefore \Delta Q = \Delta Q_{AB} + \Delta Q_{BC}$$

$$\Delta Q = 450R(\ell n4-\ln3)$$

Note : Solution is based on direct data. B and C are not satisfying the condition of isothermal process.

44. A capacitor, $C_1 = 6\mu\ F$ is charged to a potential difference of $V_0 = 5\ V$ using a 5 V battery. The battery is removed and another capacitor, $C_2 = 12\ \mu F$ is inserted in place of the battery. When the switch ' S ' is closed, the charge flows between the capacitors for some time until equilibrium condition is reached. What are the charges (q_1 and q_2) on the capacitors C_1 and C_2 when equilibrium condition is reached.

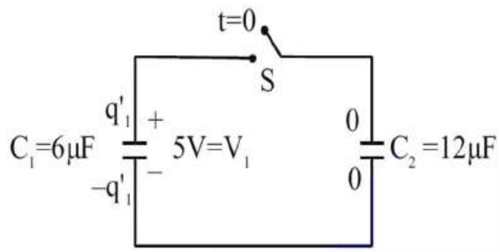


(1) $q_1 = 15\mu C, q_2 = 30\mu C$ (2) $q_1 = 30\mu C, q_2 = 15\mu C$

(3) $q_1 = 10\mu C, q_2 = 20\mu C$ (4) $q_1 = 20\mu C, q_2 = 10\mu C$

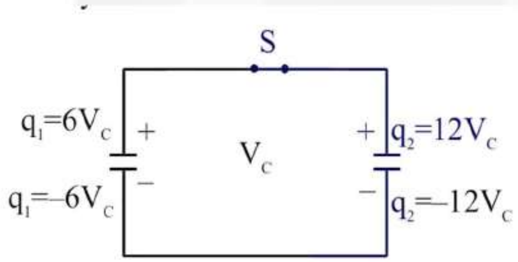
key. (3)

Sol.



$q_1^{'} = 6 \times 5 = 30\mu C$

Finally

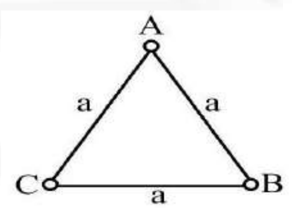


$6\ V_C + 12\ V_c = 30 + 0$

$18\ V_c = 30$

$V_C = \frac{30}{18} = \frac{5}{3}\text{Volt} \Rightarrow q_1 = \frac{6 \times 5}{3} = 10\mu C \Rightarrow q_2 = \frac{12 \times 5}{3} = 20\mu C$

45.

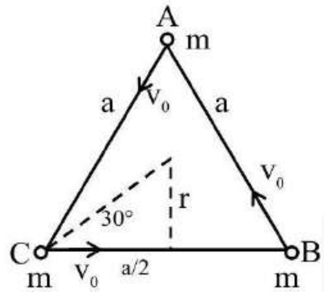


Three equal masses m are kept at vertices (A, B, C) of an equilateral triangle of side a in free space. At $t = 0$, they are given an initial velocity $\vec{V}_A = V_0\overrightarrow{AC},\ \vec{V}_B = V_0\overrightarrow{BA}$ and $\vec{V}_C = V_0\overrightarrow{CB}$. Here, $\overrightarrow{AC}, \overrightarrow{CB}$ and $\overrightarrow{BA}$ are unit vectors along the edges of the triangle. If the three masses interact gravitationally, then the magnitude of the net angular momentum of the system at the point of collision is :

(1) $\frac{1}{2}amV_0$ (2) 3 a m V (3) $\frac{\sqrt{3}}{2}amV_0$ (4) $\frac{3}{2}amV_0$

Key (3)

Sol.



$\tan 30^\circ = \frac{2r}{a} = \frac{1}{\sqrt{3}}\ r = \frac{a}{2\sqrt{3}}\quad L = (mvr_\perp)\times 3 = mv_0\frac{a}{2\sqrt{3}}\times 3 = \frac{\sqrt{3}}{2}mv_0a$

(NUMERICAL VALUE TYPE)

This section contains 10 questions. Each question is numerical value type. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to second decimal place. (e.g. 6.25, 7.00, 0.33, 30, 30.27, 127.30). Attempt any five questions out of 10. **Marking scheme: +4 for correct answer, 0 if not attempted and 0 in all other cases.**

46. Two planets, A and B are orbiting a common star in circular orbits of radii R_A and R_B, respectively, with $R_B = 2R_A$. The planet B is $4\sqrt{2}$ times more massive than planet A. The ratio $\left(\frac{L_B}{L_A}\right)$ of angular momentum (L_B) of planet B to that of planet $A(L_A)$ is closest to integer .

Key (8)

Sol. $L = mv_0R = m\sqrt{\frac{GM}{R}}R = m\sqrt{GMR}$

here M is mass of star $\frac{L_B}{L_A} = \frac{m_B}{m_A}\sqrt{\frac{R_B}{R_A}} = 4\sqrt{2}\sqrt{\frac{2}{1}}\quad \frac{L_B}{L_A} = 8$

47. A parallel plate capacitor consisting of two circular plates of radius 10 cm is being charged by a constant current of 0.15 A . If the rate of change of potential difference between the plates is 7×10^8 V/s then the integer value of the distance between the parallel plates is -

(Take, $\epsilon_0 = 9\times 10^{-12}\frac{F}{m}, \pi = \frac{22}{7}$) μm.

Key (1320)

Sol. $V = \frac{Q}{C} = \frac{it}{\left(\frac{\epsilon_0 A}{d}\right)} = \frac{itd}{\epsilon_0(\pi r^2)} \Rightarrow d = \frac{\epsilon_0(\pi r^2)}{i}\left(\frac{v}{t}\right) = \frac{(9\times 10^{-12})\left(\frac{22}{7}\right)(0.1)^2}{0.15}(7\times 10^8)\text{m}\quad d = 1320\mu\text{ m}$

48. The magnetic field inside a 200 turns solenoid of radius 10 cm is 2.9×10^{-4} Tesla. If the solenoid carries a current of 0.29 A , then the length of the solenoid is πcm.

Key (8)

Sol. Assuming long solenoid $B=\mu_0\left(\frac{N}{\ell}\right)i$

$$\ell=\frac{\mu_0 Ni}{B}=\frac{\left(4\pi\times10^{-7}\right)(200)(0.29)}{2.9\times10^{-4}}\text{ m}=8\pi\text{ cm}$$

49. A physical quantity Q is related to four observables a,b,c,d as follows : $Q=\frac{ab^4}{cd}$

where, $a=(60\pm3)\text{Pa}; b=(20\pm0.1)\text{m};$
$c=(40\pm0.2)\text{Nsm}^{-2}$ and $d=(50\pm0.1)\text{m}$,

then the percentage error in Q is $\frac{x}{1000}$,
where $x=$.

Key. 7700

Sol. $Q=\frac{ab^4}{cd}\Rightarrow\frac{\Delta Q}{Q}\times100=\left[\frac{\Delta a}{a}+4\frac{\Delta b}{b}+\frac{\Delta c}{c}+\frac{\Delta d}{d}\right]\times100$

$$\Rightarrow\frac{x}{1000}=\left[\frac{3}{60}+4\left(\frac{0.1}{20}\right)+\left(\frac{0.2}{40}\right)+\frac{0.1}{50}\right]\times100\Rightarrow x=7700$$

50. Two cars P and Q are moving on a road in the same direction. Acceleration of car P increases linearly with time whereas car Q moves with a constant acceleration. Both cars cross each other at time $t=0$, for the first time. The maximum possible number of crossing(s) (including the crossing at $t=0$) is .

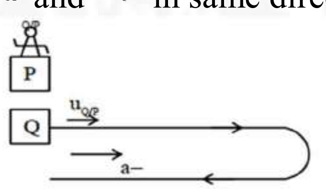
Key: (3)

Sol. $a_p=kt, k$ is constant $a_Q=a, a$ is constant $a_{QP}=a_Q-a_p=a-kt$

as initial velocities are not mentioned in question, so will have to assume two cases.

Case-I

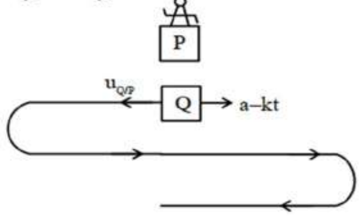
u_{or} and a_{QP} in same direction



Total number of crossing = 2

Case-II

u_{QP} and a_{QP} in opposite direction



Total number of crossing = 3

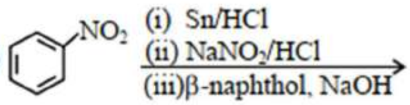
CHEMISTRY **Max Marks: 100**

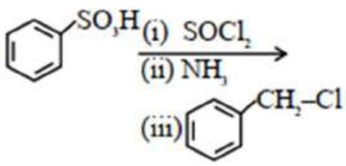
(SINGLE CORRECT ANSWER TYPE)

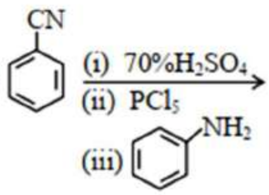
This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.

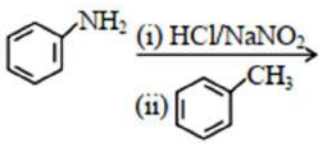
Marking scheme: +4 for correct answer, 0 if not attempted and –1 in all other cases.

51. Which one of the following reaction sequences will give an azo dye ?

(1) 

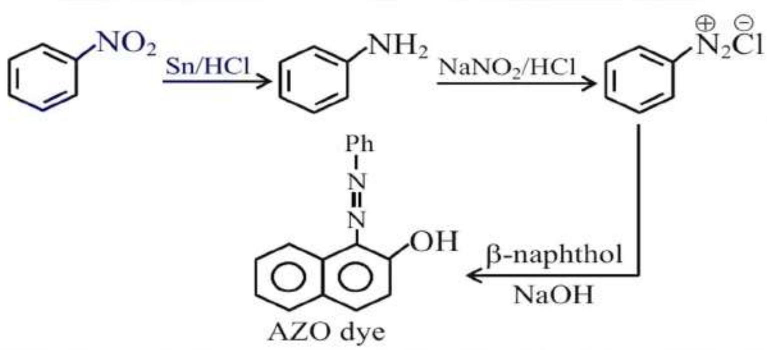
(2) 

(3) 

(4) 

Key: (1)

Sol :



52. The calculated spin-only magnetic moments of $K_3[Fe(OH)_6]$ and $K_4[Fe(OH)_6]$ respectively are :

1) 4.90 and 4.90 B.M. 2) 5.92 and 4.90 B.M.

3) 3.87 and 4.90 B.M. 4) 4.90 and 5.92 B.M

key. (2)

Sol: Conceptual

53. Consider the equilibrium

$CO(g)+3H_2(g)\rightleftharpoons CH_4(g)+H_2O(g)$ If the pressure applied over the system increases by two fold at constant temperature then

(A) Concentration of reactants and products increases.

(B) Equilibrium will shift in forward direction.

(C) Equilibrium constant increases since concentration of products increases.

(D) Equilibrium constant remains unchanged as concentration of reactants and products remain same.

Choose the correct answer from the options given below :

(1) (A) and (B) only (2) (A), (B) and (D) only

(3) (B) and (C) only (4) (A), (B) and (C) only

Key:(1)

Sol: Conceptual

54. Identify the homoleptic complexes with odd number of d electrons in the central metal.

(A) $[FeO_4]^{2-}$ (B) $[Fe(CN)_6]^{3-}$ (C) $[Fe(CN)_5NO]^{2-}$ D) $[CoCl_4]^{2-}$ (E) $\left[Co(H_2O)_3\ F_3\right]$

Choose the correct answer from the options given below.

1) (A), (B) and (D) only 2) (C) and (E) only

3) (A), (C) and (E) only 4) (B) and (D) only

Key: (4)

Sol: Conceptual

55. Match List-I with List-II :

List-I Applications		List-II Batteries/Cell	
(A)	Transistors	(I)	Anode $-Zn/Hg$; Cathode $-HgO+C$
(B)	Hearing aids	(II)	Hydrogen fuel cell
(C)	Invertors	(III)	Anode -Zn ; Cathode - Carbon
(D)	Apollo space ship	(IV)	Anode -Pb ; Cathode $-Pb\,|\,PbO_2$

Choose the correct answer from the options given below :

(1) (A)-(III), (B)-(I), (C)-(IV), (D)-(II)

(2) (A)-(III), (B)-(II), (C)-(IV), (D)-(I)

(3) (A)-(IV), (B)-(III), (C)-(II), (D)-(I)

(4) (A)-(II), (B)-(III), (C)-(IV), (D)-(I)

Key:(1)

Sol: Conceptual

56. Given below are two statements :

Statement (I): It is impossible to specify simultaneously with arbitrary precision, both the linear momentum and the position of a particle.

Statement (II) : If the uncertainty in the measurement of position and uncertainty in measurement of momentum are equal for an electron, then the uncertainty in the measurement of velocity is $\geq \sqrt{\frac{h}{\pi}} \times \frac{1}{2\text{ m}}$.

In the light of the above statements, choose the correct answer from the options given below :

(1) Statement I is true but Statement II is false.
(2) Both Statement I and Statement II are true.
(3) Statement I is false but Statement II is true.
(4) Both Statement I and Statement II are false.

Key: (2)

Sol: Conceptual

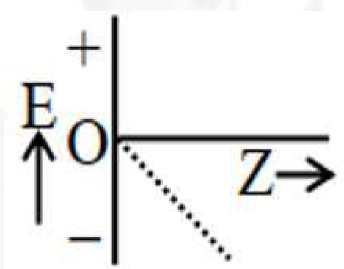
57. First ionisation enthalpy values of first four group 15 elements are given below. Choose the correct value for the element that is a main component of apatite family :

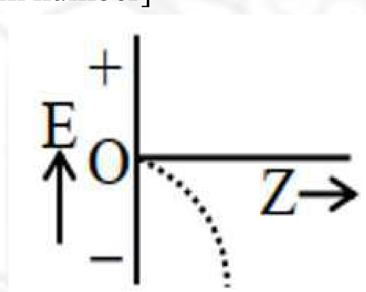
(1) 1012 kJ mol^{-1} (2) 1402 kJ mol^{-1}
(3) 834 kJ mol^{-1} (4) 947 kJ mol^{-1}

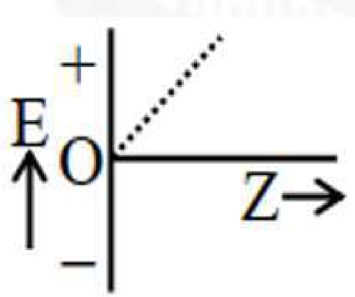
Key: (3)

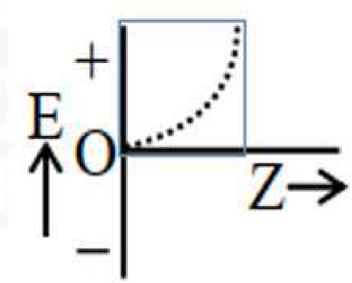
Sol: Conceptual

58. For hydrogen like species, which of the following graphs provides the most appropriate representation of E vs Z plot for a constant n ? [E: Energy of the stationary state, Z : atomic number, $n =$ principal quantum number]

1) 

2) 

3) 

4) 

Key: (2)

Sol: Conceptual

59. Given below are two statements :

Statement (I): NaCl is added to the ice at $0^{\circ}C$, present in the ice cream box to prevent the melting of ice cream.

Statement (II) : On addition of NaCl to ice at $0^{\circ}C$, there is a depression in freezing point.

In the light of the above statements, choose the correct answer from the options given below :

(1) Statement I is false but Statement II is true

(2) Both Statement I and Statement II are true

(3) Both Statement I and Statement II are false

(4) Statement I is true but Statement II is false

Key:(2)

Sol: Conceptual

60. Given below are two statements :

Statement (I) : On nitration of m-xylene with HNO_3, H_2SO_4 followed by oxidation, 4-nitrobenzene-1, 3-dicarboxylic acid is obtained as the major product.

Statement (II) : CH_3 group is o/p-directing while NO_2 group is m-directing group.

In the light of the above statements, choose the correct answer from the options given below :

(1) Both Statement I and Statement II are false

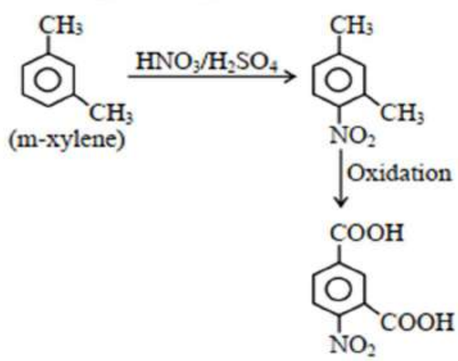
(2) Statement I is false but Statement II is true

(3) Both Statement I and Statement II are true

(4) Statement I is true but Statement II is false

Key :(3)

Sol: Statement-I



Statement-II

$-CH_3$ group is o/p directing while $-NO_2$ group is meta directing.

61. The type of oxide formed by the element among Li, Na, Be, Mg, B and Al that has the least atomic radius is :

(1) A_2O_3 (2) AO_2 (3) AO (4) A_2O

Key: (1)

Sol: Conceptual

62. O_2 gas will be evolved as a product of electrolysis of :

(A) an aqueous solution of $AgNO_3$ using silver electrodes.

(B) an aqueous solution of $AgNO_3$ using platinum electrodes.

(C) a dilute solution of H_2SO_4 using platinum electrodes.

(D) a high concentration solution of H_2SO_4 using platinum electrodes.

Choose the correct answer from the options given below :

(1) (B) and (C) only (2) (A) and (D) only
(3) (B) and (D) only (4) (A) and (C) only

Key:(1)

Sol: Conceptual

63. If C (diamond) $\rightarrow$ C (graphite) $+X\,kJmol^{-1}$

$C(\text{ diamond})+O_2(\text{ g})\rightarrow CO_2(\text{ g})+\text{Y kJ mol}^{-1}$

$C\text{ (graphite)}+O_2(\text{ g})\rightarrow CO_2(\text{ g})+\text{Z kJ mol}^{-1}$

At constant temperature. Then

(1) $X=Y+Z$ (2) $-X=Y+Z$
(3) $X=-Y+Z$ (4) $X=Y-Z$

Key: (4)

Sol: Conceptual

64. Drug X becomes ineffective after 50% decomposition. The original concentration of drug in a bottle was 16mg / mL which becomes 4 mg / mL in 12 months. The expiry time of the drug in months is _______.
Assume that the decomposition of the drug follows first order kinetics.

(1) 12 (2) 2 (3) 3 (4) 6

Key: (4)

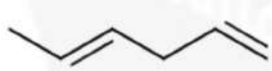
Sol: Conceptual

65. Total number of sigma (σ) and pi(π) bonds respectively present in hex-1-en-4-yne are :

(1) 13 and 3 (2) 11 and 3 (3) 3 and 13 (4) 14 and 3

Key: (1)

Sol:



σ bonds $=13$

π bonds $=3$

66. Given below are two statements :Statement (I) : In partition chromatography, stationary phase is thin film of liquid present in the inert support.
Statement (II) : In paper chromatography, the material of paper acts as a stationary phase.
In the light of the above statements, choose the correct answer from the options given below :

` (1) Both Statement I and Statement II are false

(2) Statement I is true but Statement II is false

(3) Both Statement I and Statement II are true

(4) Statement I is false but Statement II is true

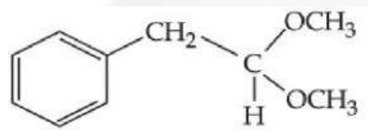
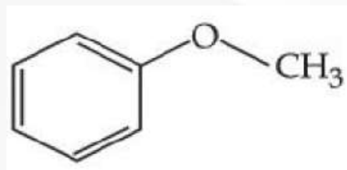
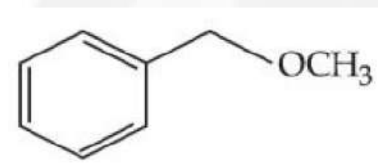
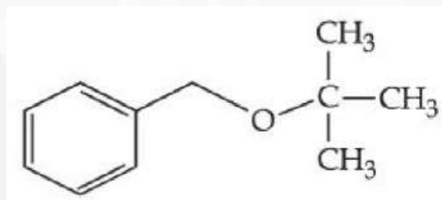
Key: (2)

Sol. Statement I is true.

In partition chromatography, stationary phase is thin liquid film present in the inert support. Statement II is false.

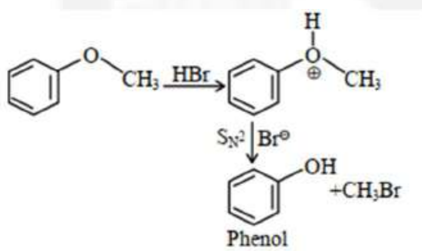
Because stationary phase in paper chromatography is water.

67. Which one of the following, with HBr will give a phenol?

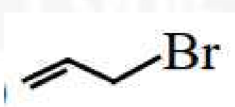
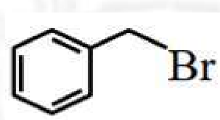
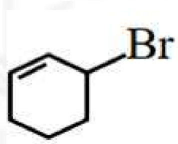
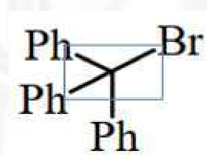
1) 
2) 
3) 
4) 

Key: (2)

Sol:

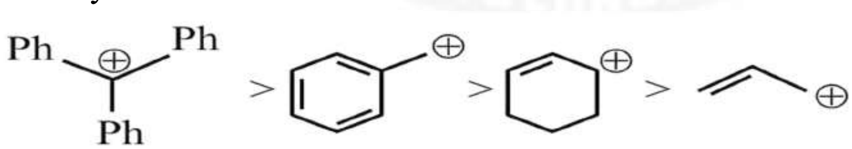


68. Which among the following halides will generate the most stable carbocation in Nucleophillic substitution reaction?

1) 
2) 
3) 
4) 

key. (4)

Sol. Stability order of carbocation



69. 0.1 M solution of KI reacts with excess of H_2SO_4 and KIO_3 solution. According to equation

$5I^- + IO_3^- + 6H^+ \rightarrow 3I_2 + 3H_2O$

Identify the correct statements :

(A) 200 mL of KI solution reacts with 0.004 mol of KIO_3

(B) 200 mL of KI solution reacts with 0.006 mol of H_2SO_4

(C) 0.5 L of KI solution produced 0.005 mol of I_2

(D) Equivalent weight of KIO_3 is equal to $\left(\frac{\text{Molecular weight}}{5}\right)$

Choose the correct answers from the options given below

1) (B) and (C) only 2) (A) and (B) only

3) (C) and (D) only 4) (A) and (D) only

Key: (4)

70. Identify the essential amino acids from below :

(A) Valine

(B) Proline

(C) Lysine

(D) Threonine

(E) Tyrosine

Choose the correct answer from the options given below:

(1) (A), (C) and (D) only (2) (A), (C) and (E) only

(3) (B), (C) and (E) only (4) (C),(D) and (E) only

Key: (1)

Sol. Valine, Lysine and Threonine are essential amino acids.

(NUMERICAL VALUE TYPE)

This section contains 10 questions. Each question is numerical value type. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to second decimal place. (e.g. 6.25, 7.00, 0.33, 30, 30.27, 127.30). Attempt any five questions out of 10.

Marking scheme: +4 for correct answer, 0 if not attempted and 0 in all other cases.

71. Consider the following low-spin complexes

$K_3\left[Co\left(NO_2\right)_6\right], K_4\left[Fe(CN)_6\right], K_3\left[Fe(CN)_6\right],$

$Cu_2\left[Fe(CN)_6\right]$ and $Zn_2\left[Fe(CN)_6\right]$.

The sum of the spin-only magnetic moment values of complexes having yellow colour is B.M. (answer is nearest integer)

Key: (o)

Sol: Conceptual

72. In the sulphur estimation, 0.20 g of a pure organic compound gave 0.40 g of barium sulphate. The percentage of sulphur in the compound is _____ $\times 10^{-1}\%$.

(Molar mass : $O = 16,\ S = 32, Ba = 137$ in $g\ mol^{-1}$)

Key: (275)

Sol: Organic Compound $BaSO_4$

0.20 gm

0.40 gm $\frac{0.40}{233}$ mol$(BaSO_4)$ $\frac{0.40}{233}$ mol (Sulphur)

$\frac{0.40}{233}\times 32$ gm (sulphur) $\quad \% S = \frac{\frac{0.40\times 32}{233}\times 100}{0.20} = 27.5\%$ or $275\times 10^{-1}\%$

73. Total number of non bonded electrons present in NO_2^- ion based on Lewis theory is.

Key: (12)

Sol: Conceptual

74. Isomeric hydrocarbons → negative Baeyer's test (Molecular formula C_9H_{12})

The total number of isomers from above with four different non-aliphatic substitution sites is –

Key: (2)

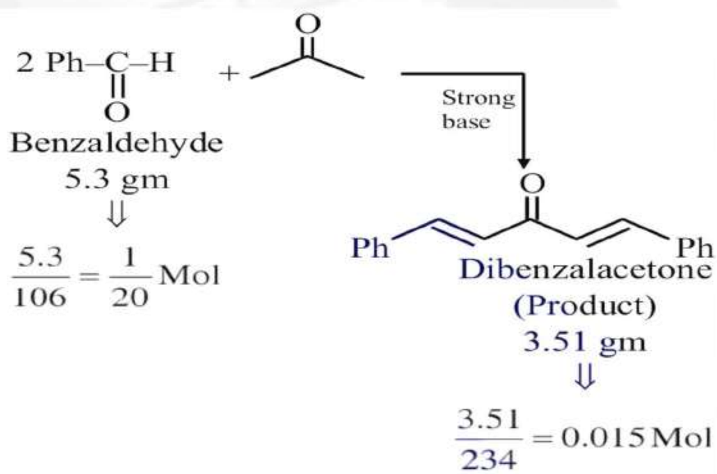
Sol :

Above two isomers of C_9H_{12} have four different sites for aromatic electrophilic substitution reaction.

75. In the Claisen-Schmidt reaction to prepare, dibenzalacetone from 5.3 g benzaldehyde, a total of 3.51 g of product was obtained. The percentage yield in this reaction was %.

Key: (6o)

Sol:



(Actual)

Theoretical $= \frac{1}{40}$ Mol

$\%$ yield $= \frac{0.015}{1/40}\times 100 \Rightarrow 60\%$

Sri Chaitanya Educational Institutions | Infinity Learn

40 Years of Excellence 1986-2025

JEE MAIN 2024

1 ALL INDIA RANK

K C BASAVA REDDY
APPL.No. 240310618179*

EMPOWERING EVERY STUDENT
TO BECOME EXTRAORDINARY

PROUDLY ACHIEVED 222 RANKS IN TOP 1000

SEIZES 4 RANKS IN TOP 10 IN ALL-INDIA RANKS

ALL INDIA RANK 3 RANK

THOTAMSETTY NIKILESH
APPL.No. 240310813888*

ALL INDIA RANK 6 RANK

HIMANSHU THALOR
APPL.No. 240310580429*

ALL INDIA RANK 9 RANK

REDDI ANIL
APPL.No.240310238514

SECURED 25 RANKS IN TOP 100
ALL INDIA OPEN CATEGORY

Sri Chaitanya - Nagpur DLP Student

Rank		Appl.No.
1	G N NIRMALKUMAR	240310385062*
9	REDDI ANIL	240310238514*
14	K C BASAVA REDDY	240310618179*
20	THOTAMSETTY NIKILESH	240310813888*
21	A V TANISH REDDY	240310807613
22	HIMANSHU THALOR	240310580429*
26	VEDANT SAINI	240310182830
28	P MEET VIKRAMBHAI	240310157524*
34	SANVI JAIN	240310150036*
40	VISHARAD SRIVASTAVA	240310046262*
43	T JAYADEV REDDY	240310167365
46	ISHAAN GUPTA	240310100229*
49	MAVURU JASWITH	240310542275*
52	DORISALA SRINIVASA REDDY	240310682440*
53	ARCHIT RAHUL PATIL	240310512311*
57	KRISHNA AGRAWAL	240310285850*
60	AYUSH GANGAL	240310270709
68	PALAGIRI SATHISH REDDY	240310905497
70	MD K GHOUSE MOHIUDDIN	240310176352
76	T V S SAI NAGA BHUSHAN	240310868568
92	M M PRUTHVI RAJ	240311084545
93	M SAI SIVA LOCHAN	240310866829*
95	RAJDEEP MISHRA	240310265621*
96	MANOJ SOHAN GAJULA	240310529661
98	KRISH NARSARIA	240310028286*

Below 100 All-India Open Category Ranks	25
Below 500 All-India Open Category Ranks	108
Below 1000 All-India Open Category Ranks	222
Below 100 All-India All Category Ranks	97
Below 1000 All Category Ranks	888

TOTAL QUALIFIED RANKS FOR JEE ADVANCED-2024 21,987

FOR OFFER ON JEE MAIN & JEE ADVANCED COURSES

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